

Minus a Negative

Why is $x - (-y) = x + y$?

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The notion that subtracting a negative is the same as adding the same (absolute) value frequently baffles students. I first saw this explanation in Mr. Bruce Lien's classroom at Adams City Middle School in Commerce City, Colorado. I've never seen this explanation by anyone else, and I think it is quite excellent - so I'm writing it down here for posterity.¹

Let's not talk numbers right off the bat, let's talk apples and oranges instead. Walk the student through the following questions, the goal is for the student to agree that you can't take oranges out of a bag (aka subtraction) if there aren't any oranges in the bag.

1. If I have a bag with 5 oranges and you take 2, how many do I have left in the bag?
2. If I have a bag with 5 oranges and you take 5, how many do I have left in the bag?
3. If I have a bag with 5 oranges and you try to take 7, how many do I have left in the bag? Do you really have 7 oranges? Or just 5?
4. If I have a bag with 5 *apples* and you try to take 7 oranges...

Let's ask the same 4 questions again but this time using +'s and -'s:

Take 2 "oranges" from a bag of 5:

$$5 - 2 = \boxed{+++++} \text{ take away } \boxed{++} = \boxed{+++}$$

Take 5 "oranges" from a bag of 5:

$$5 - 5 = \boxed{+++++} \text{ take away } \boxed{+++++} = \boxed{\text{empty}}$$

Take 7 "oranges" from a bag of 5:

$$5 - 7 = \boxed{+++++} \text{ take away } \boxed{+++++++}$$

but there is a problem here, there aren't enough +'s on the left hand side. When we were dealing with oranges, we couldn't do anything about this (except plant an orange tree). But now we are dealing with positive and negative numbers! Are the values of these two sets the same?

$$\boxed{+++++} \qquad \boxed{+++++ \begin{matrix} ++ \\ -- \end{matrix}}$$

Yes, they are. This is because two of the negatives in the right hand box cancel out two of the positives in the same box. Each box still represents 5 +'s. And *now* the right hand box has 7 +'s in it! Let's use the right hand box to solve the problem:

$$5 - 7 = \boxed{+++++ \begin{matrix} ++ \\ -- \end{matrix}} \text{ take away } \boxed{+++++++} = \boxed{--}$$

This is the familiar rule that a positive minus a *larger number* is a negative number.

¹Mr. Lien says it was in a pre-algebra text he used.

Finally, we arrive at the critical question: "take 7 oranges out of a bag of 5 apples". Again, we can't solve this problem within the world of fruit; but we can solve a similar problem in the world of positive and negative numbers.

Take -7 away from 5:

$$\begin{aligned}
 5 - (-7) &= \boxed{++++} \text{ take away } \boxed{-----} \\
 &= \boxed{++++} \boxed{++++} \text{ take away } \boxed{-----} \\
 &= \boxed{++++++++} \\
 &= 12
 \end{aligned}$$