

Triangular Numbers

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Is there a way to write an equation that says add all the numbers up to and including X ?

These numbers are called the *Triangular Numbers* and are typically written as T_n :

$$\begin{aligned}T_1 &= 1 \\T_2 &= 1 + 2 = 3 \\T_4 &= 1 + 2 + 3 + 4 = 10 \\T_n &= 1 + 2 + 3 + \cdots + n\end{aligned}$$

Mathematicians, engineers, and scientists use *summation notation* and the symbol Σ to express these summation series (they occur quite frequently in the real world!):

$$T_n = \sum_{x=1}^n x$$

Which would be expanded like this:

$$\sum_{x=1}^8 x = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$$

or if your *lower bound* is not 1:

$$\sum_{x=3}^8 x = 3 + 4 + 5 + 6 + 7 + 8 = 33$$

If the lower bound is 1, then the sum can be written analytically using Gauss' equation:

$$T_n = \frac{1}{2}n(n+1)$$

If the lower is *not* 1, then you can write:

$$\sum_{x=3}^8 x = T_8 - T_2 = \frac{1}{2}8(8+1) - \frac{1}{2}(2(2+1)) = 36 - 3 = 33$$